Section 3: Math Test — No Calculator

QUESTION 1.

Choice C is correct. Subtracting 6 from each side of $5x + 6 = 10$ yields $5x = 4$. Dividing both sides of $5x = 4$ by 5 yields $x = \frac{4}{5}$. The value of x can now be substituted into the expression $10x + 3$, giving $10\left(\frac{4}{5}\right) + 3 = 11$.

Alternatively, the expression $10x + 3$ can be rewritten as $2(5x + 6) - 9$, and 10 can be substituted for $5x + 6$, giving $2(10) - 9 = 11$.

Choices A, B, and D are incorrect. Each of these choices leads to $5x + 6 \neq 10$, contradicting the given equation, $5x + 6 = 10$. For example, choice A is incorrect because if the value of $10x + 3$ were 4, then it would follow that $x = 0.1$, and the value of $5x + 6$ would be 6.5, not 10.

QUESTION 2.

Choice B is correct. Multiplying each side of $x + y = 0$ by 2 gives $2x + 2y = 0$. Then, adding the corresponding sides of $2x + 2y = 0$ and $3x - 2y = 10$ gives $5x = 10$. Dividing each side of $5x = 10$ by 5 gives $x = 2$. Finally, substituting 2 for x in $x + y = 0$ gives $2 + y = 0$, or $y = -2$. Therefore, the solution to the given system of equations is $(2, -2)$.

Alternatively, the equation $x + y = 0$ can be rewritten as $x = -y$, and substituting x for $-y$ in $3x - 2y = 10$ gives $5x = 10$, or $x = 2$. The value of y can then be found in the same way as before.

Choices A, C, and D are incorrect because when the given values of x and y are substituted into $x + y = 0$ and $3x - 2y = 10$, either one or both of the equations are not true. These answers may result from sign errors or other computational errors.

QUESTION 3.

Choice A is correct. The price of the job, in dollars, is calculated using the expression $60 + 12nh$, where 60 is a fixed price and $12nh$ depends on the number of landscapers, n, working the job and the number of hours, h, the job takes those n landscapers. Since nh is the total number of hours of work done when n landscapers work h hours, the cost of the job increases by \$12 for each hour a landscaper works. Therefore, of the choices given, the best interpretation of the number 12 is that the company charges \$12 per hour for each landscaper.

Choice B is incorrect because the number of landscapers that will work each job is represented by n in the equation, not by the number 12. Choice C is incorrect because the price of the job increases by $12n$ dollars each hour, which will not be equal to 12 dollars unless $n = 1$. Choice D is incorrect because the total number of hours each landscaper works is equal to h. The number of hours each landscaper works in a day is not provided.

QUESTION 4.

Choice A is correct. If a polynomial expression is in the form $(x)^2 + 2(x)(y) + (y)^2$, then it is equivalent to $(x + y)^2$. Because $9a^4 + 12a^2b^2 + 4b^4 = (3a^2)^2 + 2(3a^2)(2b^2) + (2b^2)^2$, it can be rewritten as $(3a^2 + 2b^2)^2$.

Choice B is incorrect. The expression $(3a + 2b)^4$ is equivalent to the product $(3a + 2b)(3a + 2b)(3a + 2b)(3a + 2b)$. This product will contain the term $4(3a)^3 (2b) = 216a^3b$. However, the given polynomial, $9a^4 + 12a^2b^2 + 4b^4$, does not contain the term $216a^3b$. Therefore, $9a^4 + 12a^2b^2 + 4b^4 \neq (3a + 2b)^4$. Choice C is incorrect. The expression $(9a^2 + 4b^2)^2$ is equivalent to the product $(9a^2 + 4b^2)(9a^2 + 4b^2)$. This product will contain the term $(9a^2)(9a^2) = 81a^4$. However, the given polynomial, $9a^4 + 12a^2b^2 + 4b^4$, does not contain the term $81a^4$. Therefore, $9a^4 + 12a^2b^2 + 4b^4 \neq (9a^2 + 4b^2)^2$. Choice D is incorrect. The expression $(9a + 4b)^4$ is equivalent to the product $(9a + 4b)(9a + 4b)(9a + 4b) (9a + 4b)$. This product will contain the term $(9a)(9a)(9a)(9a) = 6{,}561a^4$. However, the given polynomial, $9a^4 + 12a^2b^2 + 4b^4$, does not contain the term $6{,}561a^4$. Therefore, $9a^4 + 12a^2b^2 + 4b^4 \neq (9a + 4b)^4$.

QUESTION 5.

Choice C is correct. Since $\sqrt{2k^2 + 17} - x = 0$, and $x = 7$, one can substitute 7 for x, which gives $\sqrt{2k^2 + 17} - 7 = 0$. Adding 7 to each side of $\sqrt{2k^2 + 17} - 7 = 0$ gives $\sqrt{2k^2 + 17} = 7$. Squaring each side of $\sqrt{2k^2 + 17} = 7$ will remove the square root symbol: $\left(\sqrt{2k^2 + 17}\right)^2 = (7)^2$, or $2k^2 + 17 = 49$. Then subtracting 17 from each side of $2k^2 + 17 = 49$ gives $2k^2 = 49 - 17 = 32$, and dividing each side of $2k^2 = 32$ by 2 gives $k^2 = 16$. Finally, taking the square root of each side of $k^2 = 16$ gives $k = \pm 4$, and since the problem states that $k > 0$, it follows that $k = 4$.

Since the sides of an equation were squared while solving $\sqrt{2k^2 + 17} - 7 = 0$, it is possible that an extraneous root was produced. However, substituting 4 for k in $\sqrt{2k^2 + 17} - 7 = 0$ confirms that 4 is a solution for k: $\sqrt{2(4)^2 + 17} - 7 = \sqrt{32 + 17} - 7 = \sqrt{49} - 7 = 7 - 7 = 0$.

Choices A, B, and D are incorrect because substituting any of these values for k in $\sqrt{2k^2 + 17} - 7 = 0$ does not yield a true statement.

QUESTION 6.

Choice D is correct. Since lines ℓ and k are parallel, the lines have the same slope. Line ℓ passes through the points $(-5, 0)$ and $(0, 2)$, so its slope is $\frac{0 - 2}{-5 - 0}$, which is $\frac{2}{5}$. The slope of line k must also be $\frac{2}{5}$. Since line k has slope $\frac{2}{5}$ and passes through the points $(0, -4)$ and $(p, 0)$, it follows that $\frac{-4 - 0}{0 - p} = \frac{2}{5}$, or $\frac{4}{p} = \frac{2}{5}$. Multiplying each side of $\frac{4}{p} = \frac{2}{5}$ by $5p$ gives $20 = 2p$, and therefore, $p = 10$.

Choices A, B, and C are incorrect and may result from conceptual or calculation errors.

QUESTION 7.

Choice A is correct. Since the numerator and denominator of $\frac{x^{a^2}}{x^{b^2}}$ have a common base, it follows by the laws of exponents that this expression can be rewritten as $x^{a^2-b^2}$. Thus, the equation $\frac{x^{a^2}}{x^{b^2}} = 16$ can be rewritten as $x^{a^2-b^2} = x^{16}$. Because the equivalent expressions have the common base x, and $x > 1$, it follows that the exponents of the two expressions must also be equivalent. Hence, the equation $a^2 - b^2 = 16$ must be true. The left-hand side of this new equation is a difference of squares, and so it can be factored: $(a + b)(a - b) = 16$. It is given that $(a + b) = 2$; substituting 2 for the factor $(a + b)$ gives $2(a - b) = 16$. Finally, dividing both sides of $2(a - b) = 16$ by 2 gives $a - b = 8$.

Choices B, C, and D are incorrect and may result from errors in applying the laws of exponents or errors in solving the equation $a^2 - b^2 = 16$.

QUESTION 8.

Choice C is correct. The relationship between n and A is given by the equation $nA = 360$. Since n is the number of sides of a polygon, n must be a positive integer, and so $nA = 360$ can be rewritten as $A = \frac{360}{n}$. If the value of A is greater than 50, it follows that $\frac{360}{n} > 50$ is a true statement. Thus, $50n < 360$, or $n < \frac{360}{50} = 7.2$. Since n must be an integer, the greatest possible value of n is 7.

Choices A and B are incorrect. These are possible values for n, the number of sides of a regular polygon, if $A > 50$, but neither is the greatest possible value of n. Choice D is incorrect. If $A < 50$, then $n = 8$ is the least possible value of n, the number of sides of a regular polygon. However, the question asks for the greatest possible value of n if $A > 50$, which is $n = 7$.

QUESTION 9.

Choice B is correct. Since the slope of the first line is 2, an equation of this line can be written in the form $y = 2x + c$, where c is the y-intercept of the line. Since the line contains the point (1, 8), one can substitute 1 for x and 8 for y in $y = 2x + c$, which gives $8 = 2(1) + c$, or $c = 6$. Thus, an equation of the first line is $y = 2x + 6$. The slope of the second line is equal to $\frac{1-2}{2-1}$ or -1. Thus, an equation of the second line can be written in the form $y = -x + d$, where d is the y-intercept of the line. Substituting 2 for x and 1 for y gives $1 = -2 + d$, or $d = 3$. Thus, an equation of the second line is $y = -x + 3$.

Since a is the x-coordinate and b is the y-coordinate of the intersection point of the two lines, one can substitute a for x and b for y in the two equations, giving the system $b = 2a + 6$ and $b = -a + 3$. Thus, a can be found by solving the equation $2a + 6 = -a + 3$, which gives $a = -1$. Finally, substituting -1 for a into the equation $b = -a + 3$ gives $b = -(-1) + 3$, or $b = 4$. Therefore, the value of $a + b$ is 3.

Alternatively, since the second line passes through the points (1, 2) and (2, 1), an equation for the second line is $x + y = 3$. Thus, the intersection point of the first line and the second line, (a, b) lies on the line with equation $x + y = 3$. It follows that $a + b = 3$.

Choices A and C are incorrect and may result from finding the value of only a or b, but not calculating the value of $a + b$. Choice D is incorrect and may result from a computation error in finding equations of the two lines or in solving the resulting system of equations.

QUESTION 10.

Choice C is correct. Since the square of any real number is nonnegative, every point on the graph of the quadratic equation $y = (x - 2)^2$ in the xy-plane has a nonnegative y-coordinate. Thus, $y \geq 0$ for every point on the graph. Therefore, the equation $y = (x - 2)^2$ has a graph for which y is always greater than or equal to -1.

Choices A, B, and D are incorrect because the graph of each of these equations in the xy-plane has a y-intercept at $(0, -2)$. Therefore, each of these equations contains at least one point where y is less than -1.

QUESTION 11.

Choice C is correct. To perform the division $\frac{3 - 5i}{8 + 2i}$, multiply the numerator and denominator of $\frac{3 - 5i}{8 + 2i}$ by the conjugate of the denominator, $8 - 2i$. This gives $\frac{(3 - 5i)(8 - 2i)}{(8 + 2i)(8 - 2i)} = \frac{24 - 6i - 40i + (-5i)(-2i)}{8^2 - (2i)^2}$. Since $i^2 = -1$, this can be simplified to $\frac{24 - 6i - 40i - 10}{64 + 4} = \frac{14 - 46i}{68}$, which then simplifies to $\frac{7}{34} - \frac{23i}{34}$.

Choices A and B are incorrect and may result from misconceptions about fractions. For example, $\frac{a + b}{c + d}$ is equal to $\frac{a}{c + d} + \frac{b}{c + d}$, not $\frac{a}{c} + \frac{b}{d}$. Choice D is incorrect and may result from a calculation error.

QUESTION 12.

Choice B is correct. Multiplying each side of $R = \frac{F}{N + F}$ by $N + F$ gives $R(N + F) = F$, which can be rewritten as $RN + RF = F$. Subtracting RF from each side of $RN + RF = F$ gives $RN = F - RF$, which can be factored as $RN = F(1 - R)$. Finally, dividing each side of $RN = F(1 - R)$ by $1 - R$, expresses F in terms of the other variables: $F = \frac{RN}{1 - R}$.

Choices A, C, and D are incorrect and may result from calculation errors when rewriting the given equation.

QUESTION 13.

Choice D is correct. The problem asks for the sum of the roots of the quadratic equation $12m^2 - 16m + 8 = 0$. Dividing each side of the equation by 2 gives $m^2 - 8m + 4 = 0$. If the roots of $m^2 - 8m + 4 = 0$ are s_1 and s_2, then the equation can be factored as $m^2 - 8m + 4 = (m - s_1)(m - s_2) = 0$. Looking at the coefficient of x on each side of $m^2 - 8m + 4 = (m - s_1)(m - s_2)$ gives $-8 = -s_1 - s_2$, or $s_1 + s_2 = 8$.

Alternatively, one can apply the quadratic formula to either $2m^2 - 16m + 8 = 0$ or $m^2 - 8m + 4 = 0$. The quadratic formula gives two solutions, $4 - 2\sqrt{3}$ and $4 + 2\sqrt{3}$ whose sum is 8.

Choices A, B, and C are incorrect and may result from calculation errors when applying the quadratic formula or a sign error when determining the sum of the roots of a quadratic equation from its coefficients.

QUESTION 14.

Choice A is correct. Each year, the amount of the radioactive substance is reduced by 13 percent from the prior year's amount; that is, each year, 87 percent of the previous year's amount remains. Since the initial amount of the radioactive substance was 325 grams, after 1 year, 325(0.87) grams remains; after 2 years $325(0.87)(0.87) = 325(0.87)^2$ grams remains; and after t years, $325(0.87)^t$ grams remains. Therefore, the function $f(t) = 325(0.87)^t$ models the remaining amount of the substance, in grams, after t years.

Choice B is incorrect and may result from confusing the amount of the substance remaining with the decay rate. Choices C and D are incorrect and may result from confusing the original amount of the substance and the decay rate.

QUESTION 15.

Choice D is correct. Dividing $5x - 2$ by $x + 3$ gives:

$$\begin{array}{r|l} & 5 \\ x+3 & 5x - 2 \\ & 5x + 15 \\ \hline & -17 \end{array}$$

Therefore, the expression $\frac{5x - 2}{x + 3}$ can be rewritten as $5 - \frac{17}{x + 3}$.

Alternatively, $\frac{5x - 2}{x + 3}$ can be rewritten as

$$\frac{5x - 2}{x + 3} = \frac{(5x + 15) - 15 - 2}{x + 3} = \frac{5(x + 3) - 17}{x + 3} = 5 - \frac{17}{x + 3}.$$

Choices A and B are incorrect and may result from incorrectly canceling out the x in the expression $\frac{5x-2}{x+3}$. Choice C is incorrect and may result from finding an incorrect remainder when performing long division.

QUESTION 16.

The correct answer is 3, 6, or 9. Let x be the number of \$250 bonuses awarded, and let y be the number of \$750 bonuses awarded. Since \$3000 in bonuses were awarded, and this included at least one \$250 bonus and one \$750 bonus, it follows that $250x + 750y = 3000$, where x and y are positive integers. Dividing each side of $250x + 750y = 3000$ by 250 gives $x + 3y = 12$, where x and y are positive integers. Since $3y$ and 12 are each divisible by 3, it follows that $x = 12 - 3y$ must also be divisible by 3. If $x = 3$, then $y = 3$; if $x = 6$, then $y = 2$; and if $x = 9$, then $y = 1$. If $x = 12$, then $y = 0$, but this is not possible since there was at least one \$750 bonus awarded. Therefore, the possible numbers of \$250 bonuses awarded are 3, 6, and 9. Any of the numbers 3, 6, or 9 may be gridded as the correct answer.

QUESTION 17.

The correct answer is 19. Since $2x(3x + 5) + 3(3x + 5) = ax^2 + bx + c$ for all values of x, the two sides of the equation are equal, and the value of b can be determined by simplifying the left-hand side of the equation and writing it in the same form as the right-hand side. Using the distributive property, the equation becomes $(6x^2 + 10x) + (9x + 15) = ax^2 + bx + c$. Combining like terms gives $6x^2 + 19x + 15 = ax^2 + bx + c$. The value of b is the coefficient of x, which is 19.

QUESTION 18.

The correct answer is 12. Angles ABE and DBC are vertical angles and thus have the same measure. Since segment AE is parallel to segment CD, angles A and D are of the same measure by the alternate interior angle theorem. Thus, by the angle-angle theorem, triangle ABE is similar to triangle DBC, with vertices A, B, and E corresponding to vertices D, B, and C, respectively. Thus, $\frac{AB}{DB} = \frac{EB}{CB}$ or $\frac{10}{5} = \frac{8}{CB}$. It follows that $CB = 4$, and so $CE = CB + BE = 4 + 8 = 12$.

QUESTION 19.

The correct answer is 6. By the distance formula, the length of radius OA is $\sqrt{(\sqrt{3})^2 + 1^2} = \sqrt{3+1} = 2$. Thus, $\sin(\angle AOB) = \frac{1}{2}$. Therefore, the measure of $\angle AOB$ is 30°, which is equal to $30\left(\frac{\pi}{180}\right) = \frac{\pi}{6}$ radians. Hence, the value of a is 6.

QUESTION 20.

The correct answer is $\frac{1}{4}$ or .25. In order for a system of two linear equations to have infinitely many solutions, the two equations must be equivalent.

Thus, the equation $ax + by = 12$ must be equivalent to the equation $2x + 8y = 60$. Multiplying each side of $ax + by = 12$ by 5 gives $5ax + 5by = 60$, which must be equivalent to $2x + 8y = 60$. Since the right-hand sides of $5ax + 5by = 60$ and $2x + 8y = 60$ are the same, equating coefficients gives $5a = 2$, or $a = \frac{2}{5}$, and $5b = 8$, or $b = \frac{8}{5}$. Therefore, the value of $\frac{a}{b} = \left(\frac{2}{5}\right) \div \left(\frac{8}{5}\right)$, which is equal to $\frac{1}{4}$. Either the fraction $\frac{1}{4}$ or its equivalent decimal, .25, may be gridded asthe correct answer.

Alternatively, since $ax + by = 12$ is equivalent to $2x + 8y = 60$, the equation $ax + by = 12$ is equal to $2x + 8y = 60$ multiplied on each side by the same constant. Since multiplying $2x + 8y = 60$ by a constant does not change the ratio of the coefficient of x to the coefficient of y, it follows that $\frac{a}{b} = \frac{2}{8} = \frac{1}{4}$.

Section 4: Math Test — Calculator

QUESTION 1.

Choice C is correct. Since the musician earns \$0.09 for each download, the musician earns $0.09d$ dollars when the song is downloaded d times. Similarly, since the musician earns \$0.002 each time the song is streamed, the musician earns $0.002s$ dollars when the song is streamed s times. Therefore, the musician earns a total of $0.09d + 0.002s$ dollars when the song is downloaded d times and streamed s times.

Choice A is incorrect because the earnings for each download and the earnings for time streamed are interchanged in the expression. Choices B and D are incorrect because in both answer choices, the musician will lose money when a song is either downloaded or streamed. However, the musician only earns money, not loses money, when the song is downloaded or streamed.

QUESTION 2.

Choice B is correct. The quality control manager selects 7 lightbulbs at random for inspection out of every 400 lightbulbs produced. A quantity of 20,000 lightbulbs is equal to $\frac{20{,}000}{400} = 50$ batches of 400 lightbulbs. Therefore, at the rate of 7 lightbulbs per 400 lightbulbs produced, the quality control manager will inspect a total of $50 \times 7 = 350$ lightbulbs.

Choices A, C, and D are incorrect and may result from calculation errors or misunderstanding of the proportional relationship.

QUESTION 3.

Choice A is correct. The value of m when ℓ is 73 can be found by substituting the 73 for ℓ in $\ell = 24 + 3.5m$ and then solving for m. The resulting equation is $73 = 24 + 3.5m$; subtracting 24 from each side gives $49 = 3.5m$. Then, dividing each side of $49 = 3.5m$ by 3.5 gives $14 = m$. Therefore, when ℓ is 73, m is 14.

Choice B is incorrect and may result from adding 24 to 73, instead of subtracting 24 from 73, when solving $73 = 24 + 3.5m$. Choice C is incorrect because 73 is the given value for ℓ, not for m. Choice D is incorrect and may result from substituting 73 for m, instead of for ℓ, in the equation $\ell = 24 + 3.5m$.

QUESTION 4.

Choice C is correct. The amount of money the performer earns is directly proportional to the number of people who attend the performance. Thus, by the definition of direct proportionality, $M = kP$, where M is the amount of money the performer earns, in dollars, P is the number of people who attend the performance, and k is a constant. Since the performer earns \$120 when 8 people attend the performance, one can substitute 120 for M and 8 for P, giving $120 = 8k$. Hence, $k = 15$, and the relationship between the number of people who attend the performance and the amount of money, in dollars, the performer earns is $M = 15P$. Therefore, when 20 people attend the performance, the performer earns $15(20) = 300$ dollars.

Choices A, B, and D are incorrect and may result from either misconceptions about proportional relationships or computational errors.

QUESTION 5.

Choice C is correct. If 43% of the money earned is used to pay for costs, then the rest, 57%, is profit. A performance where 8 people attend earns the performer \$120, and 57% of \$120 is $\$120 \times 0.57 = \$68.40$.

Choice A is incorrect. The amount \$51.60 is 43% of the money earned from a performance where 8 people attend, which is the cost of putting on the performance, not the profit from the performance. Choice B is incorrect. It is given that 57% of the money earned is profit, but 57% of \$120 is not equal to \$57.00. Choice D is incorrect. The profit can be found by subtracting 43% of \$120 from \$120, but 43% of \$120 is \$51.60, not \$43.00. Thus, the profit is $\$120 - \$51.60 = \$68.40$, not $\$120 - \$43.00 = \$77.00$.

QUESTION 6.

Choice B is correct. When 4 times the number x is added to 12, the result is $12 + 4x$. Since this result is equal to 8, the equation $12 + 4x = 8$ must be true. Subtracting 12 from each side of $12 + 4x = 8$ gives $4x = -4$, and then dividing both sides of $4x = -4$ by 4 gives $x = -1$. Therefore, 2 times x added to 7, or $7 + 2x$, is equal to $7 + 2(-1) = 5$.

Choice A is incorrect because -1 is the value of x, not the value of $7 + 2x$. Choices C and D are incorrect and may result from calculation errors.

QUESTION 7.

Choice D is correct. The x-intercepts of the parabola represented by $y = x^2 - 6x + 8$ in the xy-plane are the values of x for which y is equal to 0. The factored form of the equation, $y = (x - 2)(x - 4)$, shows that y equals 0 if and only if $x = 2$ or $x = 4$. Thus, the factored form, $y = (x - 2)(x - 4)$, displays the x-intercepts of the parabola as the constants 2 and 4.

Choices A, B, and C are incorrect because none of these forms shows the x-intercepts 2 and 4 as constants or coefficients.

QUESTION 8.

Choice D is correct. Since a player starts with k points and loses 2 points each time a task is not completed, the player's score will be $k - 2n$ after n tasks are not completed (and no additional points are gained). Since a player who fails to complete 100 tasks has a score of 200 points, the equation $200 = k - 100(2)$ must be true. This equation can be solved by adding 200 to each side, giving $k = 400$.

Choices A, B, and C are incorrect and may result from errors in setting up or solving the equation relating the player's score to the number of tasks the player fails to complete. For example, choice A may result from subtracting 200 from the left-hand side of $200 = k - 100(2)$ and adding 200 to the right-hand side.

QUESTION 9.

Choice A is correct. Since x is the number of 40-pound boxes, $40x$ is the total weight, in pounds, of the 40-pound boxes; and since y is the number of 65-pound boxes, $65y$ is the total weight, in pounds, of the 65-pound boxes. The combined weight of the boxes is therefore $40x + 65y$, and the total number of boxes is $x + y$. Since the forklift can carry up to 45 boxes or up to 2,400 pounds, the inequalities that represent these relationships are $40x + 65y \leq 2{,}400$ and $x + y \leq 45$.

Choice B is incorrect. The second inequality correctly represents the maximum number of boxes on the forklift, but the first inequality divides, rather than multiplies, the number of boxes by their respective weights. Choice C is incorrect. The combined weight of the boxes, $40x + 65y$, must be less than or equal to 2,400 pounds, not 45; the total number of boxes, $x + y$, must be less than or equal to 45, not 2,400. Choice D is incorrect. The second inequality correctly represents the maximum weight, in pounds, of the boxes on the forklift, but the total number of boxes, $x + y$, must be less than or equal to 45, not 2,400.

QUESTION 10.

Choice B is correct. It is given that $g(3) = 2$. Therefore, to find the value of $f(g(3))$, substitute 2 for $g(3)$: $f(g(3)) = f(2) = 3$.

Choices A, C, and D are incorrect and may result from misunderstandings about function notation.

QUESTION 11.

Choice B is correct. Tony reads 250 words per minute, and he plans to read for 3 hours, which is 180 minutes, each day. Thus, Tony is planning to read $250 \times 180 = 45{,}000$ words of the novel per day. Since the novel has 349,168 words, it will take Tony $\frac{349{,}168}{45{,}000} \approx 7.76$ days of reading to finish the novel. That is, it will take Tony 7 full days of reading and most of an 8th day of reading to finish the novel. Therefore, it will take Tony 8 days to finish the novel.

Choice A is incorrect and may result from an incorrect calculation or incorrectly using the numbers provided in the table. Choice C is incorrect and may result from taking the total number of words in the novel divided by the rate Tony reads per hour. Choice D is incorrect and may result from taking the total number of words in the novel divided by the number of pages in the novel.

QUESTION 12.

Choice D is correct. Since there were 175,000 tons of trash in the landfill on January 1, 2000, and the amount of trash in the landfill increased by 7,500 tons each year after that date, the amount of trash, in tons, in the landfill y years after January 1, 2000 can be expressed as $175{,}000 + 7{,}500y$. The landfill has a capacity of 325,000 tons. Therefore, the set of years where the amount of trash in the landfill is at (equal to) or above (greater than) capacity is described by the inequality $175{,}000 + 7{,}500y \geq 325{,}000$.

Choice A is incorrect. This inequality does not account for the 175,000 tons of trash in the landfill on January 1, 2000, nor does it accurately account for the 7,500 tons of trash that are added to the landfill each <u>year</u> after January 1, 2000. Choice B is incorrect. This inequality does not account for the 175,000 tons of trash in the landfill on January 1, 2000. Choice C is incorrect. This inequality represents the set of years where the amount of trash in the landfill is at or <u>below</u> capacity.

QUESTION 13.

Choice D is correct. Survey research is an efficient way to estimate the preferences of a large population. In order to reliably generalize the results of survey research to a larger population, the participants should be randomly selected from all people in that population. Since this survey was conducted with a population that was not randomly selected, the results are not reliably representative of all people in the town. Therefore, of the given factors, where the survey was given makes it least likely that a reliable conclusion can be drawn about the sports-watching preferences of all people in the town.

Choice A is incorrect. In general, larger sample sizes are preferred over smaller sample sizes. However, a sample size of 117 people would have allowed a reliable conclusion about the population if the participants had been selected at random. Choice B is incorrect. Whether the population is large or small, a large enough sample taken from the population is reliably generalizable if the participants are selected at random from that population. Thus, a reliable conclusion could have been drawn about the population if the 117 survey participants had been selected at random. Choice C is incorrect. When giving a survey, participants are not forced to respond. Even though some people refused to respond, a reliable conclusion could have been drawn about the population if the participants had been selected at random.

QUESTION 14.

Choice C is correct. According to the graph, the horizontal line that represents 550 billion miles traveled intersects the line of best fit at a point whose horizontal coordinate is between 2000 and 2005, and slightly closer to 2005 than to 2000. Therefore, of the choices given, 2003 best approximates the year in which the number of miles traveled by air passengers in Country X was estimated to be 550 billion.

Choice A is incorrect. According to the line of best fit, in 1997 the estimated number of miles traveled by air passengers in Country X was about 450 billion, not 550 billion. Choice B is incorrect. According to the line of best fit, in 2000 the estimated number of miles traveled by air passengers in Country X was about 500 billion, not 550 billion. Choice D is incorrect. According to the line of best fit, in 2008 the estimated number of miles traveled by air passengers in Country X was about 600 billion, not 550 billion.

QUESTION 15.

Choice A is correct. The number of miles Earth travels in its one-year orbit of the Sun is 580,000,000. Because there are about 365 days per year, the number of miles Earth travels per day is $\frac{580{,}000{,}000}{365} \approx 1{,}589{,}041$. There are 24 hours in one day, so Earth travels at $\frac{1{,}589{,}041}{24} \approx 66{,}210$ miles per hour. Therefore, of the choices given, 66,000 miles per hour is closest to the average speed of Earth as it orbits the Sun.

Choices B, C, and D are incorrect and may result from calculation errors.

QUESTION 16.

Choice B is correct. According to the table, there are 18 + 7 = 25 graduates who passed the bar exam, and 7 of them did not take the review course. Therefore, if one of the surveyed graduates who passed the bar exam is chosen at random, the probability that the person chosen did not take the review course is $\frac{7}{25}$.

Choices A, C, and D are incorrect. Each of these choices represents a different probability from the conditional probability that the question asks for. Choice A represents the following probability. If one of the surveyed graduates who passed the bar exam is chosen at random, the probability that the person chosen did take the review course is $\frac{18}{25}$. Choice C represents the following probability. If one of the surveyed graduates is chosen at random, the probability that the person chosen passed the bar exam is $\frac{25}{200}$. Choice D represents the following probability. If one of the surveyed graduates is chosen at random, the probability that the person chosen passed the exam and took the review course is $\frac{7}{200}$.

QUESTION 17.

Choice C is correct. To find the atomic weight of an unknown element that is 20% less than the atomic weight of calcium, multiply the atomic weight, in amu, of calcium by $(1 - 0.20)$: $(40)(1 - 0.20) = (40)(0.8) = 32$.

Choice A is incorrect. This value is 20% of the atomic weight of calcium, not an atomic weight 20% less than that atomic weight of calcium. Choice B is incorrect. This value is 20 amu less, not 20% less, than the atomic weight of calcium. Choice D is incorrect. This value is 20% more, not 20% less, than the atomic weight of calcium.

QUESTION 18.

Choice C is correct. The mean and median values of a data set are equal when there is a symmetrical distribution. For example, a normal distribution is symmetrical. If the mean and the median values are not equal, then the distribution is not symmetrical. Outliers are a small group of values that are significantly smaller or larger than the other values in the data. When there are outliers in the data, the mean will be pulled in their direction (either smaller or larger) while the median remains the same. The example in the question has a mean that is larger than the median, and so an appropriate conjecture is that large outliers are present in the data; that is, that there are a few homes that are valued much more than the rest.

Choice A is incorrect because a set of home values that are close to each other will have median and mean values that are also close to each other.

Choice B is incorrect because outliers with small values will tend to make the mean lower than the median. Choice D is incorrect because a set of data where many homes are valued between \$125,000 and \$165,000 will likely have both a mean and a median between \$125,000 and \$165,000.

QUESTION 19.

Choice B is correct. The median of a data set is the middle value when the data points are sorted in either ascending or descending order. There are a total of 600 data points provided, so the median will be the average of the 300th and 301st data points. When the data points are sorted in order:

- Values 1 through 260 will be 0.
- Values 261 through 450 will be 1.
- Values 451 through 540 will be 2.
- Values 541 through 580 will be 3.
- Values 581 through 600 will be 4.

Therefore, both the 300th and 301st values are 1, and hence the median is 1.

Choices A, C, and D are incorrect and may result from either a calculation error or a conceptual error.

QUESTION 20.

Choice C is correct. When survey participants are selected at random from a larger population, the sample statistics calculated from the survey can be generalized to the larger population. Since 10 of 300 students surveyed at Lincoln School have 4 siblings, one can estimate that this same ratio holds for all 2,400 students at Lincoln School. Also, since 10 of 300 students surveyed at Washington School have 4 siblings, one can estimate that this same ratio holds for all 3,300 students at Washington School. Therefore, approximately $\frac{10}{30} \times 2{,}400 = 80$ students at Lincoln School and $\frac{10}{30} \times 3{,}300 = 110$ students at Washington School are expected to have 4 siblings. Thus, the total number of students with 4 siblings at Washington School is expected to be $110 - 80 = 30$ more than the total number of students with 4 siblings at Lincoln School.

Choices A, B, and D are incorrect and may result from either conceptual or calculation errors. For example, choice A is incorrect; even though there is the same ratio of survey participants from Lincoln School and Washington School with 4 siblings, the two schools have a different total number of students, and thus, a different expected total number of students with 4 siblings.

QUESTION 21.

Choice D is correct. The difference between the number of hours the project takes, y, and the number of hours the project was estimated to take, x, is $|y - x|$. If the goal is met, the difference is less than 10, which can be represented as $|y - x| < 10$ or $-10 < y - x < 10$.

Choice A is incorrect. This inequality states that the estimated number of hours plus the actual number of hours is less than 10, which cannot be true because the estimate is greater than 100. Choice B is incorrect. This inequality states that the actual number of hours is greater than the estimated number of hours plus 10, which could be true only if the goal of being within 10 hours of the estimate were not met. Choice C is incorrect. This inequality states that the actual number of hours is less than the estimated number of hours minus 10, which could be true only if the goal of being within 10 hours of the estimate were not met.

QUESTION 22.

Choice B is correct. To rearrange the formula $I = \frac{P}{4\pi r^2}$ in terms of r^2, first multiply each side of the equation by r^2. This yields $r^2 I = \frac{P}{4\pi}$. Then dividing each side of $r^2 I = \frac{P}{4\pi}$ by I gives $r^2 = \frac{P}{4\pi I}$.

Choices A, C, and D are incorrect and may result from algebraic errors during the rearrangement of the formula.

QUESTION 23.

Choice A is correct. If I_A is the intensity measured by Observer A from a distance of r_A and I_B is the intensity measured by Observer B from a distance of r_B, then $I_A = 16I_B$. Using the formula $I = \frac{P}{4\pi r^2}$, the intensity measured by Observer A is $I_A = \frac{P}{4\pi r_A^{\,2}}$, which can also be written in terms of I_B as $I_A = 16I_B = 16\left(\frac{P}{4\pi r_B^{\,2}}\right)$. Setting the right-hand sides of these two equations equal to each other gives $\frac{P}{4\pi r_A^{\,2}} = 16\left(\frac{P}{4\pi r_B^{\,2}}\right)$, which relates the distance of Observer A from the radio antenna to the distance of Observer B from the radio antenna. Canceling the common factor $\frac{P}{4\pi}$ and rearranging the equation gives $r_B^{\,2} = 16r_A^{\,2}$. Taking the square root of each side of $r_B^{\,2} = 16r_A^{\,2}$ gives $r_B = 4r_A$, and then dividing each side by 4 yields $r_A = \frac{1}{4}r_B$. Therefore, the distance of Observer A from the radio antenna is $\frac{1}{4}$ the distance of Observer B from the radio antenna.

Choices B, C, and D are incorrect and may result from errors in deriving or using the formula $\frac{P}{4\pi r_A^{\,2}} = (16)\left(\frac{P}{4\pi r_B^{\,2}}\right)$.

QUESTION 24.

Choice A is correct. The equation of a circle with center (h, k) and radius r is $(x - h)^2 + (y - k)^2 = r^2$. To put the equation $x^2 + y^2 + 4x - 2y = -1$ in this form, complete the square as follows:

$$x^2 + y^2 + 4x - 2y = -1$$
$$(x^2 + 4x) + (y^2 - 2y) = -1$$
$$(x^2 + 4x + 4) - 4 + (y^2 - 2y + 1) - 1 = -1$$
$$(x + 2)^2 + (y - 1)^2 - 4 - 1 = -1$$
$$(x + 2)^2 + (y - 1)^2 = 4 = 2^2$$

Therefore, the radius of the circle is 2.

Choice C is incorrect because it is the square of the radius, not the radius. Choices B and D are incorrect and may result from errors in rewriting the given equation in standard form.

QUESTION 25.

Choice A is correct. In the xy-plane, the slope m of the line that passes through the points (x_1, y_1) and (x_2, y_2) is given by the formula $m = \frac{y_2 - y_1}{x_2 - x_1}$. Thus, if the graph of the linear function f has intercepts at $(a, 0)$ and $(0, b)$, then the slope of the line that is the graph of $y = f(x)$ is $m = \frac{0 - b}{a - 0} = -\frac{b}{a}$. It is given that $a + b = 0$, and so $a = -b$. Finally, substituting $-b$ for a in $m = -\frac{b}{a}$ gives $m = -\frac{b}{-b} = 1$, which is positive.

Choices B, C, and D are incorrect and may result from a conceptual misunderstanding or a calculation error.

QUESTION 26.

Choice D is correct. The definition of the graph of a function f in the xy-plane is the set of all points $(x, f(x))$. Thus, for $-4 \le a \le 4$, the value of $f(a)$ is 1 if and only if the unique point on the graph of f with x-coordinate a has y-coordinate equal to 1. The points on the graph of f with x-coordinates -4, $\frac{3}{2}$, and 3 are, respectively, $(-4, 1)$, $\left(\frac{3}{2}, 1\right)$, and $(3, 1)$. Therefore, all of the values of f given in I, II, and III are equal to 1.

Choices A, B, and C are incorrect because they each omit at least one value of x for which $f(x) = 1$.

QUESTION 27.

Choice D is correct. According to the graph, in the interval from 0 to 10 minutes, the non-insulated sample decreased in temperature by about 18°C, while the insulated sample decreased by about 8°C; in the interval from 10 to 20 minutes, the non-insulated sample decreased in temperature by about 9°C, while the insulated sample decreased by about 5°C; in the interval from 40 to 50 minutes, the non-insulated sample decreased in temperature by about 1°C, while the insulated sample decreased by about 3°C; and in the interval from 50 to 60 minutes, the non-insulated sample decreased in temperature by about 1°C, while the insulated sample decreased by about 2°C. The description in choice D accurately summarizes these rates of temperature change over the given intervals. (Note that since the two samples of water have equal mass and so must lose the same amount of heat to cool from 60°C to 25°C, the faster cooling of the non-insulated sample at the start of the cooling process must be balanced out by faster cooling of the insulated sample at the end of the cooling process.)

Choices A, B, and C are incorrect. None of these descriptions accurately compares the rates of temperature change shown in the graph for the 10-minute intervals.

QUESTION 28.

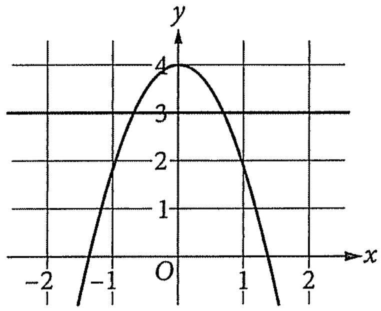
Choice B is correct. In the xy-plane, the slope m of the line that passes through the points (x_1, y_1) and (x_2, y_2) is $m = \frac{y_2 - y_1}{x_2 - x_1}$. Thus, the slope of the line through the points $C(7, 2)$ and $E(1, 0)$ is $\frac{2-0}{7-1}$, which simplifies to $\frac{2}{6} = \frac{1}{3}$. Therefore, diagonal AC has a slope of $\frac{1}{3}$. The other diagonal of the square is a segment of the line that passes through points B and D. The diagonals of a square are perpendicular, and so the product of the slopes of the diagonals is equal to -1. Thus, the slope of the line that passes through B and D is -3 because $\frac{1}{3}(-3) = -1$. Hence, an equation of the line that passes through B and D can be written as $y = -3x + b$, where b is the y-intercept of the line. Since diagonal BD will pass through the center of the square, $E(1, 0)$, the equation $0 = -3(1) + b$ holds. Solving this equation for b gives $b = 3$. Therefore, an equation of the line that passes through points B and D is $y = -3x + 3$, which can be rewritten as $y = -3(x - 1)$.

Choices A, C, and D are incorrect and may result from a conceptual error or a calculation error.

QUESTION 29.

Choice B is correct. Substituting 3 for y in $y = ax^2 + b$ gives $3 = ax^2 + b$, which can be rewritten as $3 - b = ax^2$. Since $y = 3$ is one of the equations in the given system, any solution x of $3 - b = ax^2$ corresponds to the solution $(x, 3)$ of the given system. Since the square of a real number is always nonnegative, and a positive number has two square roots, the equation $3 - b = ax^2$ will have two solutions for x if and only if (1) $a > 0$ and $b < 3$ or (2) $a < 0$ and $b > 3$. Of the values for a and b given in the choices, only $a = -2$, $b = 4$ satisfy one of these pairs of conditions.

Alternatively, if $a = -2$ and $b = 4$, then the second equation would be $y = -2x^2 + 4$. The graph of this quadratic equation in the xy-plane is a parabola with y-intercept (0, 4) that opens downward. The graph of the first equation, $y = 3$, is the horizontal line that contains the point (0, 3). As shown below, these two graphs have two points of intersection, and therefore, this system of equations has exactly two real solutions. (Graphing shows that none of the other three choices produces a system with exactly two real solutions.)



Choices A, C, and D are incorrect and may result from calculation or conceptual errors.

QUESTION 30.

Choice A is correct. The regular hexagon can be divided into 6 equilateral triangles of side length a by drawing the six segments from the center of the regular hexagon to each of its 6 vertices. Since the area of the hexagon is $384\sqrt{3}$ square inches, the area of each equilateral triangle will be $\frac{384\sqrt{3}}{6} = 64\sqrt{3}$ square inches.

Drawing any altitude of an equilateral triangle divides it into two 30°-60°-90° triangles. If the side length of the equilateral triangle is a, then the hypotenuse of each 30°-60°-90° triangle is a, and the altitude of the equilateral triangle will be the side opposite the 60° angle in each of the 30°-60°-90° triangles. Thus, the altitude of the equilateral triangle is $\frac{\sqrt{3}}{2}a$, and the area of the equilateral triangle is $\frac{1}{2}(a)\left(\frac{\sqrt{3}}{2}a\right) = \frac{\sqrt{3}}{4}a^2$. Since the area of each equilateral triangle is $64\sqrt{3}$ square inches, it follows that $a^2 = \frac{4}{\sqrt{3}}(64\sqrt{3})$ $= 256$ square inches. And since the area of the square with side length a is a^2, it follows that the square has area 256 square inches.

Choices B, C, and D are incorrect and may result from calculation or conceptual errors.

QUESTION 31.

The correct answer is 14. Since the coastal geologist estimates that the country's beaches are eroding at a rate of 1.5 feet every year, they will erode by $1.5x$ feet in x years. Thus, if the beaches erode by 21 feet in x years, the equation $1.5x = 21$ must hold. The value of x is then $\frac{21}{1.5} = 14$. Therefore, according to the geologist's estimate, it will take 14 years for the country's beaches to erode by 21 feet.

QUESTION 32.

The correct answer is 7. There are 60 minutes in each hour, and so there are $60h$ minutes in h hours. Since h hours and 30 minutes is equal to 450 minutes, it follows that $60h + 30 = 450$. This equation can be simplified to $60h = 420$, and so the value of h is $\frac{420}{60} = 7$.

QUESTION 33.

The correct answer is 11. It is given that the function $f(x)$ passes through the point (3, 6). Thus, if $x = 3$, the value of $f(x)$ is 6 (since the graph of f in the xy-plane is the set of all points $(x, f(x))$). Substituting 3 for x and 6 for $f(x)$ in $f(x) = 3x^2 - bx + 12$ gives $6 = 3(3)^2 - b(3) + 12$. Performing the operations on the right-hand side of this equation gives $6 = 3(9) - 3b + 12 = 27 - 3b + 12 = 39 - 3b$. Subtracting 39 from each side of $6 = 39 - 3b$ gives $-33 = -3b$, and then dividing each side of $-3b = -33$ by -3 gives the value of b as 11.

QUESTION 34.

The correct answer is 105. Let D be the number of hours Doug spent in the tutoring lab, and let L be the number of hours Laura spent in the tutoring lab. Since Doug and Laura spent a combined total of 250 hours in the tutoring lab, the equation $D + L = 250$ holds. The number of hours Doug spent in the lab is 40 more than the number of hours Laura spent in the lab, and so the equation $D = L + 40$ holds. Substituting $L + 40$ for D in $D + L = 250$ gives $(L + 40) + L = 250$, or $40 + 2L = 250$. Solving this equation gives $L = 105$. Therefore, Laura spent 105 hours in the tutoring lab.

QUESTION 35.

The correct answer is 15. The amount, a, that Jane has deposited after t fixed weekly deposits is equal to the initial deposit plus the total amount of money Jane has deposited in the t fixed weekly deposits. This amount a is given to be $a = 18t + 15$. The amount she deposited in the t fixed weekly deposits is the amount of the weekly deposit times t; hence, this amount must be given by the term $18t$ in $a = 18t + 15$ (and so Jane must have deposited 18 dollars each week after the initial deposit). Therefore, the amount of Jane's original deposit, in dollars, is $a - 18t = 15$.

QUESTION 36.

The correct answer is 32. Since segments LM and MN are tangent to the circle at points L and N, respectively, angles OLM and ONM are right angles. Thus, in quadrilateral $OLMN$, the measure of angle O is $360° - (90° + 60° + 90°) = 120°$. Thus, in the circle, central angle O cuts off $\frac{120}{360} = \frac{1}{3}$ of the circumference; that is, minor arc $\overset{\frown}{LN}$ is $\frac{1}{3}$ of the circumference. Since the circumference is 96, the length of minor arc $\overset{\frown}{LN}$ is $\frac{1}{3} \times 96 = 32$.

QUESTION 37.

The correct answer is 3284. According to the formula, the number of plants one year from now will be $3000 + 0.2(3000)\left(1 - \frac{3000}{4000}\right)$, which is equal to 3150. Then, using the formula again, the number of plants two years from now will be $3150 + 0.2(3150)\left(1 - \frac{3150}{4000}\right)$, which is 3283.875. Rounding this value to the nearest whole number gives 3284.

QUESTION 38.

The correct answer is 7500. If the number of plants is to be increased from 3000 this year to 3360 next year, then the number of plants that the environment can support, K, must satisfy the equation $3360 = 3000 + 0.2(3000)\left(1 - \frac{3000}{K}\right)$. Dividing both sides of this equation by 3000 gives $1.12 = 1 + 0.2\left(1 - \frac{3000}{K}\right)$, and therefore, it must be true that $0.2\left(1 - \frac{3000}{K}\right) = 0.12$, or equivalently, $1 - \frac{3000}{K} = 0.6$. It follows that $\frac{3000}{K} = 0.4$, and so $K = \frac{3000}{0.4} = 7500$.